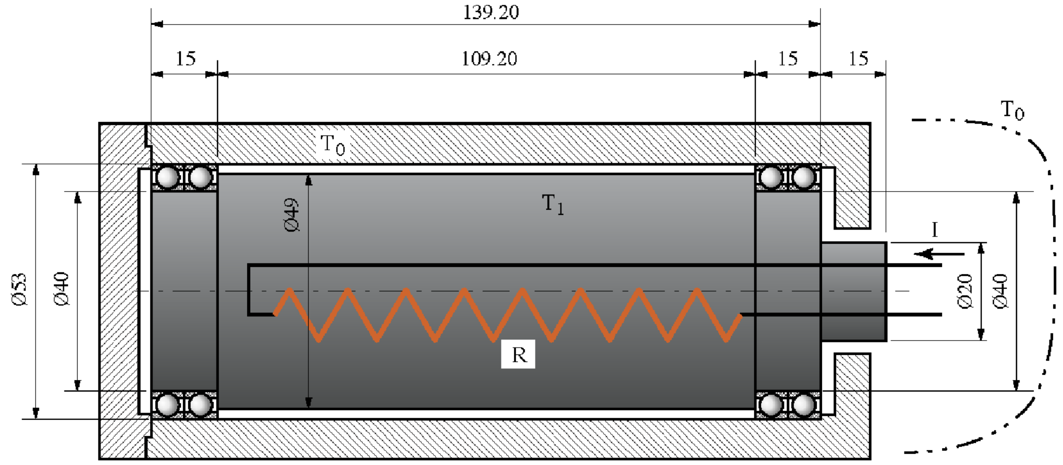
EE-580 Space Mechanisms

Exercise 4.4 - Thermal Equilibrium

v.01

Problem Statement

The thermal model of a mechanism is given by the following picture



The temperatures of the axis (T_1) and of the housing (T_0) are considered to be homogeneous and in steady state equilibrium.
A heater with an electrical resistance R is mounted on the axis.
The axis and the housing are in Aluminum with an Alodine coating (Emittance = 0.15).
The temperature is in steady state.
The thermal conductivity of a ball-bearing is 0.013 W/K.

Questions

With an electrical resistance of 5 Ω and a current of 1 A. calculate the temperature of the axis, taking into account that the housing is maintained at a constant temperature of 35°C:

- In the case where the axis and the housing are considered as black bodies
- Optional: in the case where the housing and axis are in aluminum (surface treatment Alodine 1200, emittance $\epsilon = 0.15$)

Hint: for the last question take a look at doc. [4.6] in the Moodle (Radiation Heat Transfer Between Planar Surfaces http://web.mit.edu/16.unified/www/FALL/thermodynamics/notes/node136.html)

Solution

Preliminary comments

1. The heat flux in the system results from the conductive flux through the ball-bearings in parallel with the radiative flux between the axis and the housing.

2. The axis and housing can be considered as grey body because of the material and coating, this means that they do not emit like a black body. Their emittance is lower than 1. In this case, the radiative flux is less than if they were back bodies. In consequence the axis has to heat more in order dissipate the energy coming from the heater.

   This problem can be solved for a black body by using the well-known formula presented in the course:

$$P_{rad,black} = \sigma \cdot A_{rad} \cdot (T_1^4 - T_0^4)$$

   With:

σ = 5.67 W/(m^2·K^{-4}):	Stefan-Boltzman constant
$P_{rad,black}$:	radiative power of the back body of radiative surface A_{rad}
T_0, T_1:	resp. temperatures of the two facing radiative surfaces

   For a grey body, the solution requires a more elaborated calculation taking into account the absorption/emission and reflection properties of the materials.
   See http://web.mit.edu/16.unified/www/FALL/thermodynamics/notes/node136.html ([4.7] on MOODLE) for a demonstration of the following formula for the radiative power of grey bodies a and b.

$$P_{rad,grey} = \frac{\sigma \cdot A_{rad} \cdot (T_1^4 - T_0^4)}{\frac{1}{\varepsilon_a} + \frac{1}{\varepsilon_b} - 1}$$

   With:

ε_a, ε_b:	resp. emittance of body a and b

   Hence the solution of the problem becomes, adding the heat conduction through the ball-bearings:

$$P = n_b \cdot k \cdot (T_1 - T_0) + \frac{\sigma \cdot A_{rad} \cdot (T_1^4 - T_0^4)}{\frac{1}{\varepsilon_a} + \frac{1}{\varepsilon_b} - 1}$$

   With:

n_b:	number of ball-bearings
k:	thermal conductivity of a single ball-bearing [W/K]

Numerical solution

Stefan-Boltzman constant [W/(m ^{2}K ^{4})]:

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In[ ]:= σ = 5.67 × 10^-8;
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Diameter of the axis [m]:

In[ ]:= $d_a = 49 \times 10^{-3};$

Radiative length of the axis [m]:

In[ ]:= $L = 109.2 \times 10^{-3};$

Rotor diameter at bearing [m]:

In[ ]:= $d_b = 40 \times 10^{-3};$

Coupling diameter [m]:

In[ ]:= $d_c = 20 \times 10^{-3};$

Length of the coupling axis [m]:

In[ ]:= $l_c = 15 \times 10^{-3};$

Radiative surface [m ^{2}]:

In[ ]:= $A_{rad} = \pi \left(d_a L + 2 \frac{d_b^2}{4} + d_c l_c \right)$

Out[ ]=

0.0202658

Thermal conductivity of a ball-bearing [W/K]:

In[ ]:= $k_b = 0.013;$

Number of ball-bearings:

In[ ]:= $n_b = 4;$

Emittance of the axis (material):

In[ ]:= $\epsilon_a = 0.16;$

Emittance of the housing (material):

In[ ]:= $\epsilon_b = 0.16;$

Housing temperature [K]:

In[ ]:= $T_0 = 273.15 + 35$

Out[ ]=

308.15

Heating current [A]:

In[ ]:= $I_h = 1;$

Dissipative electrical resistance [Ω]:

In[ ]:= $R_h = 5;$

Dissipated power [W]:

In[∘]:= $P_h = I_h^2 R_h$

Out[∘]=

5

Solving the power formula (eq. cf. here above)

Solving (numerically) the power dissipation as a function of T [W] in order to get the temperature of the axis T $_1$ [K]:

In[∘]:= $$\text{NSolve}\left[n_b\, k_b\, (T_1 - T_0) + \frac{\sigma\, A_{rad}\, \left(T_1^4 - T_0^4\right)}{\frac{1}{\epsilon_a} + \frac{1}{\epsilon_b} - 1} == P_h \,\&\&\, T_1 > 0,\, T_1,\, \text{Reals}\right]$$

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Out[∘]=

{{T_1 → 381.098}}

Note: Only the positive solution makes sense for the temperature

Specific cases:

Axis emittance:

In[∘]:= $\epsilon_a = 0.16;$

Housing emittance (black body):

In[∘]:= $\epsilon_b = 1;$

In[∘]:= $$\text{NSolve}\left[n_b\, k_b\, (T_1 - T_0) + \frac{\sigma\, A_{rad}\, \left(T_1^4 - T_0^4\right)}{\frac{1}{\epsilon_a} + \frac{1}{\epsilon_b} - 1} == P_h \,\&\&\, T_1 > 0,\, T_1,\, \text{Reals}\right]$$

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Out[∘]=

{{T_1 → 369.954}}

Axis emittance (black body):

In[∘]:= $\epsilon_a = 1;$

Housing emittance (black body):

In[∘]:= $\epsilon_b = 1;$

In[∘]:= $$\text{NSolve}\left[n_b\, k_b\, (T_1 - T_0) + \frac{\sigma\, A_{rad}\, \left(T_1^4 - T_0^4\right)}{\frac{1}{\epsilon_a} + \frac{1}{\epsilon_b} - 1} == P_h \,\&\&\, T_1 > 0,\, T_1,\, \text{Reals}\right]$$

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Out[∘]=

{{T_1 → 332.726}}